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AN ELECTROHYDRODYNAMIC METHOD OF RETARDING THE TRANSITION OF A BOUNDARY LAYER*

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The possibility of a downstream displacement of the point of transition of a laminar boundary layer to the turbulent mode, as a result of electrohydrodynamic (EHD) action on the boundary layer flow is considered. A method based on using the electrostatic volume forces appearing when a charged medium flows in an electric field, may turn out to be one of the novel, effective and economic methods of controlling the boundary layer /1/. The assessment of the result of EHD action on the position of the transition point is obtained below using the results of a calculation of the spatial amplification factors of small perturbations of the Tomlin-Schlichting wave type in the EHD boundary layer, and the e^n-method of predicting the transition /2/.

1. Consider the flow of a viscous incompressible fluid past a semi-infinite dielectric plate with relative permittivity ε_w, with the flow velocity denoted by u_∞. The coordinate system chosen has its origin at the leading edge of the plate, the x axis is directed along the surface parallel to the flow velocity vector, and the y axis is perpendicular to the surface. It is assumed that semi-infinite grid electrodes Γ_1 and Γ_2, not affecting the gas flow (Fig.1), are erected on the plane perpendicularly to the direction of the oncoming flow. The distance between the electrodes is l, and their dimensionless coordinates are x_1 and x_2. The earthed electrode Γ_2 is an ion collector, and the emitter electrode Γ_1 simulates the unipolar charge sources situated upstream /3/. An electrode Γ_3, modelling the electrode used to impart a definite form to the ionic flow is placed inside the plate, parallel to its surface, at a distance y_3 between Γ_1 and Γ_2.

It is assumed that $x_1 \leqslant O(1)$, so that the Reynolds number determined over the length l is characteristic for the boundary layer between the electrodes.

The system of electrogasdynamic equations describing the steady flow of a viscous incompressible gas with unipolar charge, has the following form in dimensionless coordinates /3, 4/:

$$\left(\frac{\partial\psi}{\partial y}\frac{\partial}{\partial x}-\frac{\partial\psi}{\partial x}\frac{\partial}{\partial y}-\varepsilon^2\nabla^2\right)\nabla^2\psi = N\left(E_x\frac{\partial q}{\partial y}-E_y\frac{\partial q}{\partial x}\right) \tag{1.1}$$

$$\left(\frac{\partial\psi}{\partial y}+E_x\right)\frac{\partial q}{\partial x}-\left(\frac{\partial\psi}{\partial x}-E_y\right)\frac{\partial q}{\partial y}+q^2=\frac{\varepsilon^2}{\lambda}\nabla^2 q \tag{1.2}$$

$$\nabla\times\mathbf{E}=0,\quad \nabla\mathbf{E}=q$$

Here ψ is the hydrodynamic stream function, $\mathbf{E}=(E_x, E_y)$ is the electric field vector, q is the volume charge density, $\mathrm{Re}=u_\infty l/\nu$ is the Reynolds number, $\varepsilon=\mathrm{Re}^{-1/2}$, $\lambda=\nu/D$ is the ratio of the kinematic viscosity of the gas to the ion diffusion coefficient, $N=\varepsilon_0/(\rho b^2)$ is the EHD interaction parameter, ρ and ε_0 are the density and absolute permittivity and b is the ionic mobility.

If we use a corona discharge as a source of unipolar charge, then $\lambda\sim 1$ [5], $N\sim 10^{-3}$ [3]. In this case we can write, for the values of the Reynolds number ranging from 10^6 to 10^7, $N=k\varepsilon$, where $k=O(1)$.

The set of equations (1.1), (1.2) can be solved using the following boundary conditions. For the stream function we have the conditions of adhesion to the plate surface and a uniform stream at infinity. The electrical parameters in the interelectrode region are found by specifying, on the latter, the electric potential distribution. We specify on the emitter the initial volume charge density distribution. As $y\to\infty$, the component E_x of the

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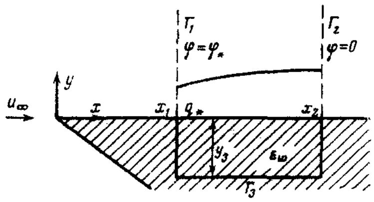

Fig.1

electric field vector is constant between Γ_1 and Γ_2. The following conditions must hold on the dielectric surface of the plate:

$$E_{1x}=E_{2x},\quad E_{1y}+\varepsilon_u E_{2y}=\sigma,\quad E_{1y}q-\frac{\varepsilon^2}{\lambda}\frac{\partial q}{\partial y}=0 \tag{1.3}$$

where the indices 1 and 2 refer, respectively, to the region of flow and the region within the plate, and σ is the surface charge density. Below we shall consider a dielectric surface not absorbing an electric charge, and in this case we shall have $\sigma=0$.

In addition, the values of the charge at the collector, also as $y\to\infty$, must in general be given for the last elliptic equation of (1.2). Should we however not plan to investigate the structure of the diffusion layer near the collector /5/, which falls outside the scope of this paper, then as we shall show later, there will be no need to specify these conditions.

2. The solution of (1.1)–(1.2) is obtained in the form of expansions in terms of the small parameter ε /6/.

In the region with characteristic dimensions $x\sim1$, $y\sim1$, the gas velocity is given in the basic approximation by $u=\partial\psi/\partial y=1$, $v=-\partial\psi/\partial x=0$, and the electric parameters are described by the system of equations

$$\nabla^2\varphi=-q,\quad \left(1-\frac{\partial\varphi}{\partial x}\right)\frac{\partial q}{\partial x}-\frac{\partial\varphi}{\partial y}\frac{\partial q}{\partial y}+q^2=0 \tag{2.1}$$

within the region of flow, and by the Laplace equation

$$\nabla^2\varphi=0 \tag{2.2}$$

within the plate, where we have introduced the electric potential φ.

To construct a solution of system (2.1), (2.2) satisfying, apart from the boundary conditions, conditions (1.3), we must consider the ionic diffusion layer on the dielectric surface of the plate appearing as a result of the last condition of (1.3). The layer thickness depends obviously on the electric field component E_y exerting the pressure on the ionic flow. The magnitude of this component can be controlled by means of the electrode Γ_3, either by varying the potential distribution on it, or the distance y_3. The magnitude of the accelerating field E_x is basically determined by the potential difference between Γ_1 and Γ_2. When the accelerating field potential is close to its breakdown value and the velocity of flow $u_\infty\sim100$ m/sec we have $E_x\sim1$. Different solutions of the problem can be constructed, depending on the magnitude of E_y.

Let $E_y\sim\varepsilon^n$, where $n<1$ is a parameter. The last condition of (1.3) yields an estimate for the diffusion layer thickness $\delta_q\sim\varepsilon^{2-n}$, in which we use the independent variables x, $Y=\lambda\varepsilon^{n-2}y$ and the expansions

$$\begin{aligned}
&E_x(x,y;\varepsilon)=E_{x0}^\circ(x,Y)+\varepsilon^2E_{x1}^\circ(x,Y)+\ldots\\
&E_y(x,y;\varepsilon)=\varepsilon^nE_{y0}^\circ(x,Y)+\varepsilon^{2-n}E_{y1}^\circ(x,Y)+\ldots\\
&q(x,y;\varepsilon)=\varepsilon^{2n-2}Q_0(x,Y)+Q_1(x,Y)+\ldots\\
&u(x,y;\varepsilon)=\varepsilon^{1-n}u_0(x,Y)+\ldots,\ v(x,y;\varepsilon)=\\
&\quad\varepsilon^{3-2n}v_0(x,Y)+\ldots
\end{aligned} \tag{2.3}$$

Substituting (2.3) into (1.2) we obtain a set of equations for the ionic diffusion layer in the main approximation

$$\frac{\partial E_{x0}^\circ}{\partial Y}=0,\quad \lambda\frac{\partial E_{y0}^\circ}{\partial Y}=Q_0,\quad E_{y0}^\circ\frac{\partial^2E_{y0}^\circ}{\partial Y^2}+\left(\frac{\partial E_{y0}^\circ}{\partial Y}\right)^2=\frac{\partial^3E_{y0}^\circ}{\partial Y^3} \tag{2.4}$$

When the characteristic charge density in the corona discharge is 10^{-5} K/m^3, $l\sim1$ m, $u_\infty>100$ m/sec, we have in the external region of the flow $q\sim1$ and the expansions

$$\begin{aligned}
&E_x(x,y;\varepsilon)=E_{x0}(x,y)+\ldots,\ E_y(x,y;\varepsilon)=\\
&\quad\varepsilon^nE_{y0}(x,y)+\ldots\\
&q(x,y;\varepsilon)=q_0(x,y)+\ldots
\end{aligned} \tag{2.5}$$

Using the last two conditions of (1.3) and matching (2.3) with (2.5), we obtain the following boundary conditions for the last equation of (2.4):

$$Y=0:\ E_{y0}^\circ\frac{\partial E_{y0}^\circ}{\partial Y}=\frac{\partial^2E_{y0}^\circ}{\partial Y^2}\qquad E_{y0}^\circ=\varepsilon_wE_{y0}^{(2)}\qquad Y\to\infty:\ \frac{\partial E_{y0}^\circ}{\partial Y}\to0 \tag{2.6}$$

where $E_{y0}^{(2)}$ is the principal term of the expansion of the electric field inside the plate, of the form (2.5).

In addition, the matching of (2.3) with (2.5) yields the conditions

$$E_{x0}^{\circ}(x) = E_{x0}(x, 0), \quad E_{y0}^{\circ}(x, \infty) = E_{y0}(x, 0) \tag{2.7}$$

The solution of equations (2.4) with boundary conditions (2.6) has the form

$$E_{y0}^{\circ}(x, Y) = E_{\infty}\frac{1+\bar\beta}{1-\bar\beta}, \quad Q_0(x, Y) = 2\lambda E_{\infty}^2 \frac{\bar\beta}{(1-\bar\beta)^2} \tag{2.8}$$

$$E_{\infty}(x) = E_{y0}^{\circ}(x, \infty), \quad \beta(x) = \frac{\varepsilon_w E_{y0}^{(2)}(x, 0) - E_{\infty}}{\varepsilon_w E_{y0}^{(2)}(x, 0) + E_{\infty}}$$

$$\bar\beta(x, Y) = \beta(x)\exp(E_{\infty} Y)$$

The function $E_{y0}^{(2)}(x, 0)$, appearing in the solution (2.8) is found from the first condition of (1.3), and to find $E_{\infty}(x)$ we must consider the following approximation in the diffusion layer. For the functions E_{y1}° and Q_1 we obtain the problem

$$\frac{\partial E_{x0}^{\circ}}{\partial x} + \lambda\frac{\partial E_{y1}^{\circ}}{\partial Y} = Q_1 \tag{2.9}$$

$$E_{x0}^{\circ}\frac{\partial Q_0}{\partial x} + \lambda\left(E_{y0}^{\circ}\frac{\partial Q_1}{\partial Y} + \frac{\partial Q_0}{\partial Y}E_{y1}^{\circ}\right) + 2Q_0Q_1 = \lambda\frac{\partial^2 Q_1}{\partial Y^2} \tag{2.10}$$

$$Y = 0: \; E_{y1}^{\circ} = \varepsilon_w E_{y1}^{(2)}, \quad E_{y0}^{\circ}Q_1 + Q_1 E_{y1}^{\circ} = \frac{\partial Q_1}{\partial Y} \tag{2.11}$$

$$Y \to \infty: \; Q_1 \to q_0(x, 0)$$

Integrating Eq.(2.10) is Y from 0 to ∞ and in x from x_1 to x, and taking into account (2.9), (2.6), (2.7), (2.11), we obtain the relation

$$E_{x0}(x, 0)\int_0^{\infty} Q_0(x, Y)\,dY = -\lambda\int_{x_1}^{x} q_0(x, 0)\,E_{y0}(x, 0)\,dx \tag{2.12}$$

Equation (2.12) has a simple physical meaning. The electric current in the diffusion layer passing across a section x, is equal to the current flowing to this cross-section from a direction perpendicular to the plate surface along the outer, diffusion-free region. Using (2.12) we obtain the following integral equation for determining the function $E_{\infty}(x) = E_{y0}(x, 0)$

$$E_{y0}(x, 0) = \varepsilon_w E_{y0}^{(2)}(x, 0) - E_{x0}^{-1}(x, 0)\int_{x_1}^{x} q_0(x, 0)\,E_{y0}(x, 0)\,dx \tag{2.13}$$

The electric parameters of the flow were computed using the following algorithm. A difference mesh was constructed between the electrodes, and certain values of the function $E_{y0}^{(2)}(x,0)$ were assigned at it nodes on the plate surface. A method of consecutive approximations was used to solve the system of equations (2.1) /7/. A value of the potential was specified at the electrodes Γ_1 and Γ_2, and the initial charge density on Γ_1 as $y \to \infty$. A distribution $\partial\varphi/\partial y = -\varepsilon^n E_{y0}(x, 0)$ was taken at the boundary $y = 0$ where the function $E_{y0}(x, 0)$ was given by (2.13) whose right-hand side contains the values of this function obtained from the previous iteration and $E_{y0}(x, 0) = \varepsilon_w E_{y0}^{(2)}(x, 0)$ is taken from the first iteration. Equation (2.2) was then solved for the given distribution $\partial\varphi(x, 0)/\partial y = -\varepsilon^n E_{y0}^{(2)}(x, 0)$ and potentials of the electrodes $\Gamma_1, \Gamma_2, \Gamma_3$. The procedure was repeated for the second specified distribution $E_{y0}^{(2)}(x, 0)$, and its correct values were determined at each node of the difference mesh by the method of secants using the first condition of (1.3).

The functions $E_{\infty}(x) = E_{y0}(x, 0)$, $E_{y0}^{(2)}(x, 0)$ computed in this manner make it possible to determine the electric field and the charge distribution in the diffusion layer (2.8).

3. Having calculated the electric parameters in the main approximation, we can find the perturbations of hydrodynamic functions caused by the EHD action. As a result of substituting the expansions for the stream function $\psi(x, y; \varepsilon) = y + \varepsilon\psi_2(x, y) + \ldots$ and electric parameters (2.5) into (1.1), we obtain the second approximation problem in the region of inviscid flow, taking both the thickness of the boundary layer displacement /6/ and EHD action into account (the parameter k was defined above)

$$\nabla^2\left(\frac{\partial\psi_2}{\partial x}\right) = kE_{x0}\frac{\partial q_0}{\partial y} \tag{3.1}$$

$$\psi_2(x,0)=\begin{cases} 0, & x<0 \\ -1{,}72/\sqrt{2x}, & x>0 \end{cases} \tag{3.2}$$

where $\psi_2(x, y) = o(y)$ in the oncoming flow.

Since problem (3.1), (3.2) is linear, its solution can be sought in the form of the superposition $\psi_2 = \psi_{20} + \psi_{21}$, where the function ψ_{20} satisfies the Laplace equation and boundary conditions (3.2) and is known /6/, while ψ_{21} satisfies Eq.(3.1) with zero boundary conditions.

Consider the boundary layer flow using the new variables $x, z = y/\varepsilon$ and expansions for the stream function $\psi(x, y; \varepsilon) = \varepsilon\chi_1(x, z) + \varepsilon^2\chi_2(x, z) + \ldots$, substitution of which into (1.1) leads to the following equation for χ_1:

$$\frac{\partial^3\chi_1}{\partial z^3} + \frac{\partial\chi_1}{\partial x}\frac{\partial^2\chi_1}{\partial z^2} - \frac{\partial\chi_1}{\partial z}\frac{\partial^2\chi_1}{\partial x\,\partial z} = 0 \tag{3.3}$$

The boundary conditions for this equation are determined by matching χ_1 with the solutions in the region of inviscid flow and in the diffusion layer. We shall show that these conditions have the usual form

$$\frac{\partial\chi_1}{\partial z}(x, \infty) = 1, \quad \chi_1(x, 0) = \frac{\partial\chi_1}{\partial z}(x, 0) = 0 \tag{3.4}$$

We have the following equation and a single boundary condition for the second approximation, taking the conditions of matching at the outer boundary of the boundary layer into account:

$$\frac{\partial^3\chi_2}{\partial z^3} + \frac{\partial\chi_1}{\partial x}\frac{\partial^2\chi_2}{\partial z^2} - \frac{\partial\chi_1}{\partial z}\frac{\partial^2\chi_2}{\partial x\partial z} + \frac{\partial^2\chi_1}{\partial z^2}\frac{\partial\chi_2}{\partial x} - \frac{\partial^2\chi_1}{\partial x\,\partial z}\frac{\partial\chi_2}{\partial z} = -\frac{\partial^2\psi_2}{\partial x\partial y}(x, 0) \tag{3.5}$$

$$\frac{\partial\chi_2}{\partial z}(x, \infty) = \frac{\partial\psi_2}{\partial y}(x, 0) \tag{3.6}$$

The solution in the diffusion layer will be obtained for the case $n = {}^2/_3$.

Using the diffusion layer variables $x, Y = \varepsilon^{-4/3} y$ and substituting the expansion (2.3) and $\psi = \varepsilon^{5/3}\Psi_1(x, Y) + \varepsilon^{7/3}\Psi_2 + \ldots$ into (1.1), we obtain

$$\frac{\partial^4\Psi_1(x, Y)}{\partial Y^4} = 0, \quad \frac{\partial^4\Psi_2(x, Y)}{\partial Y^4} = -kE_{x0}(x, 0)\frac{\partial Q_0(x, Y)}{\partial Y}$$

which on integration yields, together with the expressions (2.8) and conditions of adhesion,

$$\frac{\partial\Psi_1}{\partial Y} = A_1(x)Y^2 + A_2(x)Y \tag{3.7}$$

$$\frac{\partial\Psi_2}{\partial Y} = A_3(x)Y^2 + \left[\frac{2k\beta}{1-\beta}E_\infty E_{x0}(x, 0) + A_4(x)\right]Y +$$

$$2k\lambda^{-1}E_{x0}(x, 0)\ln\frac{1-\beta\exp(\lambda E_\infty Y)}{1-\beta}$$

Matching these expressions with the solution in the boundary layer, we find the unknown functions appearing in (3.7) and boundary conditions for the equations (3.3), (3.5).

$$A_1(x) = 0, \quad A_2(x) = \frac{\partial^2\chi_1}{\partial z^2}(x, 0), \quad A_3(x) = 0 \tag{3.8}$$

$$A_4(x) = -\frac{2k\beta(x)}{1-\beta(x)}E_\infty(x)E_{x0}(x, 0)$$

$$\chi_1(x, 0) = \frac{\partial\chi_1}{\partial z}(x, 0) = \chi_2(x, 0) = 0$$

$$\frac{\partial\chi_2}{\partial z}(x, 0) = -2k\lambda^{-1}E_{x0}(x, 0)\ln(1-\beta)$$

Changing now to the Blasius variables x, $\eta = z/\sqrt{x}$, $\chi_1 = \sqrt{x}f_1(\eta)$, $\chi_2 = \sqrt{x}f_2(x, \eta)$, and using (3.3)–(3.8), we obtain the following problems in the boundary layer (a prime denotes differentiation with respect to η):

$$f_1''' + \frac{1}{2}f_1f_1'' = 0, \quad f_1(0) = f_1'(0) = 0, \quad f_1'(\infty) = 1$$

$$f_2''' + \frac{1}{2}(f_1f_2'' + f_1''f_2) = x\left[f_1\frac{\partial f_2'}{\partial x} - f_1''\frac{\partial f_2}{\partial x} - \frac{\partial^2\psi_2}{\partial x\,\partial y}(x, 0)\right]$$

$$f_2(x, 0) = 0, \; f_2'(x, 0) = -2k\lambda^{-1}E_{x0}(x, 0)\ln(1-\beta)$$

$$f_2'(x, \infty) = \partial\psi_2(x, 0)/\partial y$$

Equally useful two-term expansions for the longitudinal velocity and the curvature of its profile have the following form in the boundary layer:

$$f'(x,\eta) = f_1'(\eta) + \varepsilon\{f_2'(x,\eta) + 2k\lambda^{-1}E_{x0}\ln[1 - \beta\exp(\lambda E_\infty \varepsilon^{-1/2}\sqrt{x}\eta)]\} \tag{3.9}$$

$$f'''(x,\eta) = f_1'''(\eta) - \varepsilon^{1/2}\, 2k\lambda\beta x E_{x0} E_\infty^2 \exp(\lambda E_\infty \varepsilon^{-1/2}\sqrt{x}\eta) \times [1 - \beta\exp(\lambda E_\infty \varepsilon^{-1/2}\sqrt{x}\eta)]^{-2} + \varepsilon f_2'''(x,\eta) \tag{3.10}$$

4. Equations describing the development of small perturbations on the electrohydrodynamic boundary layer are derived, as in the linear theory of stability, by linearizing the complete system of electrohydrodynamic equations and the corresponding boundary conditions. Here we can also show that to a first approximation the perturbations in the electric flow parameter do not occur in the equations describing the perturbations of the hydrodynamic parameters. Thus the stability of the boundary EHD layer can be studied within the framework of the analyis of the Orr-Sommerfeld equation.

It should be noted that although the terms in the boundary layer equations connected with the non-parallelism of the flow are of the same order as the terms governed by the EHD action, nevertheless, since the problem of stability is linear, the principle of superposition holds and the effects mentioned can be taken into account independently.

The two-dimensional perturbations of the stream function $\psi(x, y, t)$ are sought in the form /8/

$$\psi = v(\eta)\exp\left\{i\left[\mathrm{Re}_l^{1/2}\int_{x_0}^{x} x^{-1/2}\alpha(x)\,dx - tFR\right]\right\} \tag{4.1}$$

and this leads to the Orr-Sommerfeld eigenvalue problem

$$v^{IV} - 2\alpha^2 v'' + \alpha^4 v = iR[(\alpha f' - FR)(v'' - \alpha^2 v) - \alpha f''' v] \tag{4.2}$$
$$v(0) = v'(0) = 0;\ v \to 0,\ v' \to 0 \quad \text{as} \quad \eta \to \infty$$

Here η is the Blasius variable introduced above, $R = (x\,\mathrm{Re}_l)^{1/2}$, t is the time, $\alpha(x) = \alpha_r(x) + i\alpha_i(x)$ is a complex function, $F = \omega\nu/u_\infty^2$ is a frequency parameter, ω is the perturbation frequency, x_0 is the coordinate of a certain fixed cross-section, and the functions $f'(x,\eta), f'''(x,\eta)$ are given by expressions (3.9), (3.10).

Problem (4.2) was solved by reducing it to a Cauchy problem using the procedure of orthogonalization /9/.

The position of the transition point of the boundary later where it changes from the laminar to the turbulent mode, was estimated by calculating the spatial amplification factors of small perturbations /10/ representing, at $\omega = \text{const}$, the ratio of the perturbation amplitude at the point x, to its magnitude at x_0 of the lower branch of the neutral stability curve

$$a = \exp\left[-\int_{x_0}^{x}\left(\frac{\mathrm{Re}_l}{x}\right)^{1/2}\alpha_i\,dx\right] = \exp\left(-2\int_{R_0}^{R}\alpha_i\,dR\right)$$

Using the solution of problem (4.2) we determine, in each cross-section x, the values of the rate of growth of the perturbation α_i as a function of the frequency parameter F. Next we construct an envelope of the curves $\ln a = f(\mathrm{Re}_x)$, where $\mathrm{Re}_x = x\,\mathrm{Re}_l$, which determines the values of the maximum growth coefficients a_m. Adopting a definite value of $\ln a_m$ at the transition point, we can find the Reynolds number Re_x of the transition.

5. We have carried out the computations for the following values of the parameters of the problem: $\mathrm{Re}_l = 1.5\cdot 10^6$, $K = 1$, $\varepsilon_w = 3$, $\lambda = 0.3$, $x_1 = 2/3$, $y_3 = 0.5$. The potential $\varphi_* = 3$ is specified on the whole emitter Γ_1, and the initial charge density $q_* = 3$ on its segment $0 \leqslant y \leqslant 0.05$. The potential distribution on the control electrode Γ_3 was given in the form

$$\varphi(x, -y_3) = \varphi_*\{(x - x_2) + A\sin[\pi(x - x_2)]\},\quad A = 0.1$$

The effect of such parameters as the electrode potentials and initial charge density q_*, on the result of the EHD action on the boundary layer, is of interest. The role of the parameter φ_* is clear: as the emitter potential increases, the magnitude of the accelerating field $E_{x0}(x, 0)$ increases approximately in proportion and the EHD action increases. The effect of the charge density manifest itself through the variation in the function $\beta(x)$, appearing in (3.9), (3.10). Using (2.8), (2.13), we obtain the following expression for this function:

$$\beta = \int_{x_1}^{x} q_0(x,0)E_{y_0}(x,0)\,dx \Big/ \left[2\varepsilon_w E_{y0}^{(2)}(x,0)E_{x0}(x,0) - \int_{x_1}^{x} q_0(x,0)E_{y0}(x,0)\,dx\right]$$

It is clear that the values of $\beta(x)$, increase as the charge density increases, and this also enhances the EHD action. The effect of the magnitude of the compressing field $E_{y0}(x, 0)$

(the parameter A) is contradictory. On the one hand, increasing $E_{y0}(x, 0)$, leads, according to (3.10), to an increase in the absolute magnitude of the curvature of the velocity profile in the boundary layer, on the other hand the thickness of the region in which the increase in the curvature occurs is reduced, and this weakens the result of the EHD action. Computations have shown that the optimal mean value of the function $E_{y0}(x, 0)$ is approximately -1, and we have the corresponding value $A = 0,1$.

It whould be noted that, according to the computations carried out, the charge gradients $\partial q_0/\partial y$, appearing in (3.1) are small within the ion stream, of the order of $10^{-2} - 10^{-3}$. For this reason the values of the function $\partial\psi/\partial x$, obtained by solving (3.1), and of the functions $\partial\psi_2(x, 0)/\partial y$, $\partial^2\psi_2(x, 0)/\partial x\partial y$ appearing in the boundary layer problem, are mush less than unity. During the calculations the functions were assumed to be equal to zero.

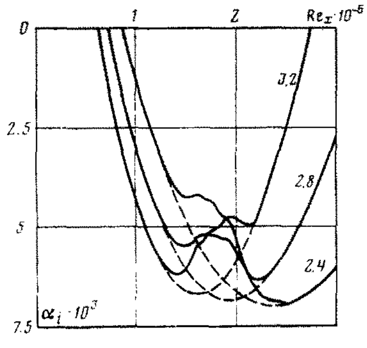

Fig.2

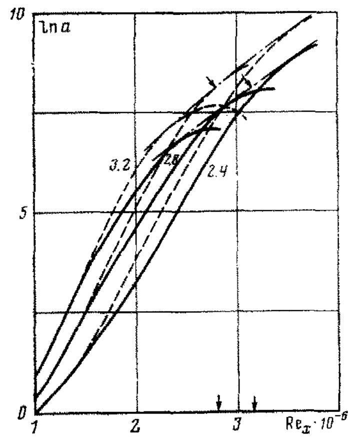

Fig.3

Figs.2 and 3 shows the results of the calculations. Fig.2 shows the dependence of the perturbation growth rate α_i on the local Reynolds number Re_x, and Fig.3 shows the same relations for the function $\ln a$. The dashed line in these figures show the corresponding relations for the Blasius flow, and the solid lines show the results of the calculations including the EHD contribution. The numbers indicate the values of the parameter $F \cdot 10^5$, for which the computations were carried out. The range $(1 - 2.5)\cdot 10^6$ of Reynolds number corresponded to the interelectrode gap.

According to /11/, the transition on the flat plate takes place at $Re_x = 2.8 \cdot 10^6$. By determining the value of $\ln a_m$, corresponding to this value of the Reynolds number in the case of Blasius flow, we obtain the following value of the transition Reynolds number for the EHD boundary layer: $Re_x = 3.16 \cdot 10^6$. Thus, according to our calculations the EHD action can be expected to lead to an approximately 13% increase in the value of the Reynolds number.

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ON CERTAIN CONSERVATION PROPERTIES IN GAS DYNAMICS*

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A previously unknown invariant of the vertex lines of a stationary barotropic, ideal gas flow is discovered, An analogue of this invariant and of other invariants of the stream and vortex lines is obtained for the more general case of non-barotropic flow.

An equation is obtained describing the variation in the projection of the vorticity on the direction of the velocity in three-dimensional ideal gas flow. Examples are shown where the projection does not vary along the stream lines, and this yields an additional integral of the gas-dynamic equations.

1. Consider the steady flow of an ideal compressible gas. We denote the velocity vector by $\mathbf{v}$, $\boldsymbol{\omega} = \operatorname{rot} \mathbf{v}$ is the vorticity, p is the pressure and ρ is the density. In gas-dynamics the quantities conserved along the stream lines (stream line invariants) are of interest. We known, in particular, that along with the entropy σ the Ertel vortex potential $E_0 = (\boldsymbol{\omega}\cdot\nabla\sigma)/\rho$ is also conserved along the stream lines.

In a barotropic gas flow /1—3/ $E_\lambda = (\boldsymbol{\omega}\cdot\nabla\lambda)/\rho$ serves as the stream line invariant

$$\mathbf{v}\cdot\nabla\left(\frac{\boldsymbol{\omega}\cdot\nabla\lambda}{\rho}\right) = 0 \tag{1.1}$$

where λ is an arbitrary function, constant along the stream lines

$$\mathbf{v}\cdot\nabla\lambda = 0 \tag{1.2}$$

Relations (1.1), (1.2) express the Euler-Ertel theorem /1/ for a compressible barotropic gas.

We must also establish the invariants of the vortex lines. The Bernoulli function H represents one of these invariants:

$$\boldsymbol{\omega}\cdot\nabla H = 0,\quad H = \frac{q^2}{2} + \int\frac{dp}{\rho(p)},\quad q^2 = \mathbf{v}\cdot\mathbf{v} \tag{1.3}$$

We find that the relations are definitely commutative with respect to interchange of the vectors $\mathbf{v}$ and $\boldsymbol{\omega}$. This yields a new invariant of the vortex lines and is expressed by the following theorem.

Theorem. Let μ be a twice continuously differentiable function constant along the vortex lines of the continuous barotropic gas flow

$$\boldsymbol{\omega}\cdot\nabla\mu = 0 \tag{1.4}$$

Then the quantity $\theta_\mu = \mathbf{v}\cdot\nabla\mu$ will also remain constant along the vortex lines

$$\boldsymbol{\omega}\cdot\nabla(\mathbf{v}\cdot\nabla\mu) = 0 \tag{1.5}$$

To prove the theorem we transform the left side of expression (1.5) using the well-known formula for the gradient of a scalar product. We obtain

$$\boldsymbol{\omega}\cdot\nabla(\mathbf{v}\cdot\nabla\mu) = \boldsymbol{\omega}\cdot(\mathbf{v}\cdot\nabla)\nabla\mu + \boldsymbol{\omega}\cdot(\nabla\mu\cdot\nabla)\,\mathbf{v} \tag{1.6}$$

Applying the operator $\mathbf{v}\cdot\nabla$ to (1.4), we reduce the first term on its right-hand side to the form

$$\boldsymbol{\omega}\cdot(\mathbf{v}\cdot\nabla)\nabla\mu = -\nabla\mu\cdot(\mathbf{v}\cdot\nabla)\,\boldsymbol{\omega} \tag{1.7}$$

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